GEOPHYSICAL RESEARCH LETTERS, VOL. 17, NO 11, PAGES 1841-1844, OCTOBER 1990

LOW-DIMENSIONAL CHAOS IN MAGNETOSPHERIC ACTIVITY FROM AE TIME SERIES

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Abstract. The magnetospheric response to the solar wind input, as represented by the time series measurements of the AE index, has been examined using phase space reconstruction techniques. The system was found to behave as a low-dimensional chaotic system with a fractal dimension of 3.6 and has a Kolmogorov entropy <0.2/min. These are indicative that the dynamics of the system can be adequately described by four independent variables and a corresponding intrinsic time scale is of the order of 5 min. The relevance of the results to magnetospheric modelling is discussed.

Introduction

The earth's magnetosphere is a complex, nonlinear dynamical system which responds in a relatively unpredictable fashion to variations of the solar wind energy input. There is evidence [Gonzalez et al., 1989] that the southward component of the interplanetary magnetic field (IMF) is a key parameter that controls the solar wind input. Many indices, e.g., AE, AU, AL, D_{st}, K_p, etc. are used to characterize the magnetospheric response to this solar wind input. Each one measures a different type of response [Mayaud, 1980; Baumjohann, 1986]. The auroral electrojet (AE) index is a measure of the horizontal current strength flowing in the lower ionosphere. It is often taken as the substorm index. Many empirical and physical models attempt to study the response of AE to the IMF variations [Clauer et al., 1983; Bargatze et al., 1985; Kamide and Slavin, 1986] using linear prediction filter techniques. Most recently, Tsurutani et al. [1990] examined the Fourier transform of the AE time series and correlated it to the IMF spectrum. The results of these studies indicate: i) the absence of periodic or quasiperiodic behavior. Rather, the power is always concentrated in the lowest frequency suggesting an aperiodic behavior (either deterministic-chaotic or random); ii) a 1/f power spectrum, similar to the IMF spectrum, at low frequencies ($f<6.10^{-5}$Hz) with a break near 6.10^{-5} Hz followed by a $1/f^{2.2-2.4}$ spectrum. Namely, the magnetosphere is a low-pass filter and its internal dynamics controls the high frequency behavior. From the microphysics point of view the "internal dynamics" is composed of a complex of interactions which involve phenomena such as modifications in the ionospheric conductivity, cross-tail current disruption or tearing mode instabilities, field-aligned currents, anomalous resistivity or double layers, etc. Although progress in understanding these phenomena has been made, we are still far from producing quantitative predictive models of the interaction [Butler and Papadopoulos, 1984]. The techniques described above revealed the presence of multiple time scales but could not assess the dynamics responsible for the observed output. It is the purpose of this letter to propose a different type of analysis and modelling of the data that describe the magnetospheric response based on recent developments in the study of phase spaces of nonlinear systems [Mayer-Kress, 1986].

Until recently nonlinear systems with aperiodic behavior, such as the magnetosphere, were described in terms of power spectra or correlation functions. However, while spectral studies are suitable for the study and classification of periodic, quasiperiodic or random systems, they are unable to provide meaningful information for a wide class of systems known today as "chaotic" systems. In such systems the broadband spectra and "random" behavior apparent in snapshots or time series, are the consequences of aperiodic deterministic motion with extreme sensitivity to initial conditions rather than stochastic behavior. In the past few years techniques have been developed which allow us to derive quantities associated with the phase-space evolution of the system and its associated geometry. These quantities are known as dimensions, entropies, Lyapounov exponents and singularity spectra [Farmer et al., 1983; Mayer-Kress, 1986]. A virtue of these quantities is that they provide simple, global and topologically invariant information. The dimension, for example, with which we deal mainly in this paper, is a single-number information on the system. It represents the minimum number of independent variables that can describe the system. Furthermore, from the point of view of analyzing experimental data, these quantities have the virtue that they can be calculated easily from time series even from a single dynamic variable. In this paper we present an analysis of time series of the AE index using nonlinear dynamical techniques. The analysis demonstrates that magnetospheric behavior as represented by the AE index is a low-dimensional attractor and thus amenable to further dynamical analysis. In view of the novelty of such techniques in the space physics community we present in the next section a somewhat extended description of the time-series analysis technique.

Nonlinear Time-Series Analysis

A dissipative system, such as the magnetosphere, has the property that its phase space volume contracts as the system approaches its asymptotic state. This dynamical state is called the *attractor* and may generally be described by fewer variables than the original system (here the independent variables correspond to the degrees of freedom of the system). If we consider the attractor as a set of points embedded in the phase space then by a well-defined procedure we can assign to it a number called its dimension. The dimension turns out to be a lower bound in the number of independent variables necessary to describe or model the attractor. If

Paper number 90GL01887
0094-8276/90/90GL-01887$03.00

this positive-definite dimension is fractional (in which case the attractor is a so-called *fractal* set) then the bound is the next highest integer. Also a fractional dimension is indicative of chaotic dynamics governing the motion on the attractor (quasi- or periodic motion is revealed by integer dimension). The presence and degree of chaos in the system can then be quantified by different diagnostics, such as the Kolmogorov entropy.

The dimension and entropy are found from the system's evolution in phase space. Experimentally we often have access to time series of only one or few variables, but this obstacle can be overcome if the system variables are sufficiently coupled. In such cases the time delay embedding technique [Takens, 1981] is an appropriate method for using the time series data to reconstruct the phase space and obtain its characteristic quantities.

Following this method we construct an m-component "state" vector X_i from a time series x(t) as

$$X_i = \{x_1(t_i), x_2(t_i), \ldots x_m(t_i)\}$$

where $x_k(t_i) = x(t_i + (k-1)\tau)$ and τ is an appropriate time delay (of the order of characteristic physical time scales). In this reconstructed phase space the distribution of state vectors is directly related to the sought dimension. By defining a suitable quantity that depends on the distribution and examining its scaling with distance in phase space, one can extract the value of the dimension. Here we use the *correlation integral* [Grassberger and Procaccia 1983a] defined for N vectors distributed in an m-dimensional space as a function of distance r:

$$C(r;m) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Theta(r - |X_i - X_j|)$$

where Θ is the Heavyside step function. If the number of points is large enough, as assumed above, this distribution will obey a power-law scaling with r for small r: $C(r;m) \sim r^{\nu}$, where ν is the *correlation dimension*, defined as

$$\nu = \lim_{r\to 0} \frac{\log C(r;m)}{\log r}$$

As we increase our control parameter m, the correlation dimension is seen to converge to its true value. Generally $\nu \leq m$ with the equality holding when there is no attractor and the system explores the available state space, as in the case of random or "noisy" systems with many degrees of freedom.

From the correlation integrals it is straightforward to determine the K_2 entropy [Grassberger and Procaccia, 1983b]:

$$K_2 = \lim_{r\to 0} \frac{1}{\tau} \ln \frac{C(r;m)}{C(r;m+1)}$$

where τ is the sampling rate. This entropy is a lower bound of the Kolmogorov entropy which measures the rate of loss of information, or difference of evolution between almost identical initial conditions. When K_2 is finite, the Kolmogorov entropy is nonzero, and the system is chaotic; if K_2 is infinite the system is random (nondeterministic). The inverse of this entropy is a timescale over which we can accurately predict the behavior of the system.

AE index and Magnetospheric activity

As mentioned in the introduction the AE index is a convenient characterization of the magnetospheric activity often interpreted as a measure of substorm activity [Baumjohann, 1986]. In this paper, time series of AE data [Allen, 1987] were used to study the nonlinear dynamical properties of magnetospheric activity using the above methods. These data reflect the activity observed during the first 21 days of January 1983 averaged over 1 minute intervals, a total of 30 kmin. We chose the sampling time small enough to be able to resolve the time scales of substorms and related magnetospheric phenomena. The time series is shown in Figure 1 and contains several distinct periods of various activity levels. These data were examined in segments of 5 kmin long to ensure homogeneous activity, except for the most active segment at 21–27 kmin, which was examined as a whole. A level of activity was assigned to each segment using the integral occurence percentage [Bargatze et al., 1985]. Concatenated pairs of successive segments were compared to determine the variation of results with the segment length N.

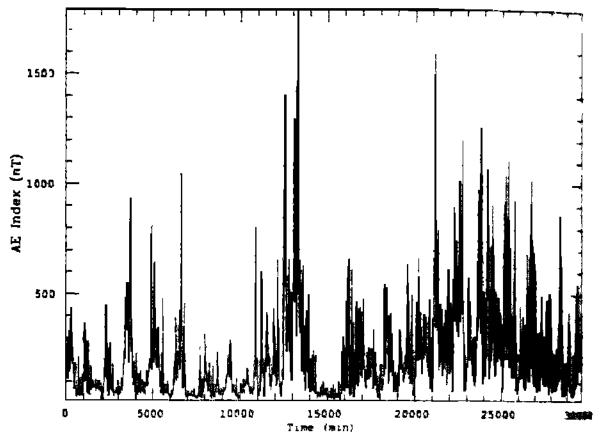

Fig. 1. The time series of the AE index for 1– 21 January 1983, with 1 min. resolution.

Each data set was embedded in a reconstructed state space using the method of time delays [Takens, 1981]. After observing that results varied little with the time delay (in the range of 5 to 30 minutes) τ was fixed to 10 min. In the m-dimensional state space the correlation integral [Grassberger and Procaccia, 1983a] is formed and plotted against r in a log-log diagram. The correlation dimension can then be calculated in different ways as the slope of the log(C)-log(r) curve. Our favorite was a least squares fit, although other methods of linear regression gave very similar results. For a chaotic system as m is raised ν grows and then saturates. Indeed ν always converged rapidly for the AE data to a number between 3 and 4 (see Figure 2). Figure 3 gives a plot of attractor dimension versus activity, each point corresponding to a different data segment. All points are contained between 3 and 4, independently of activity, with an average dimension 3.6. Longer data sets (the most active set, 21–27 kmin, and the concatenated sets) also fall in the same range. However, for a few segments the activity was particularly nonuniform, e.g. the interval 15–20 kmin (Figure 1). As a result the observed attractor is deformed and not populated with enough points. Unless the number of points is increased correlations are poor and there is no convergence to a low dimension value as the data look random to the correlation algorithm.

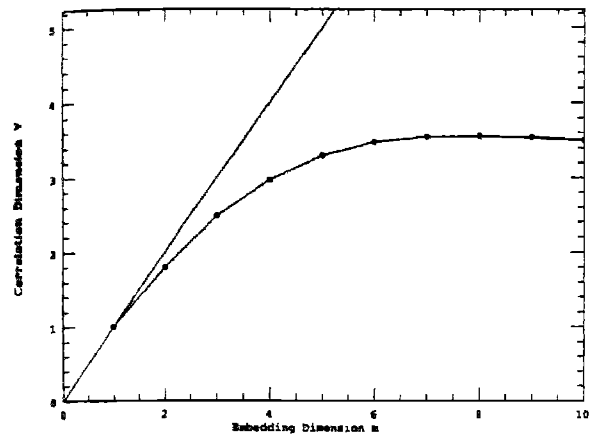

Fig. 2. The attractor dimension ν is seen to saturate (here to 3.6) as the dimension m of the embedding space is increased. The data come from the 0–5 kmin segment of the data in Fig. 1. The straight line corresponds to the dimension exhibited by a random or noisy system.

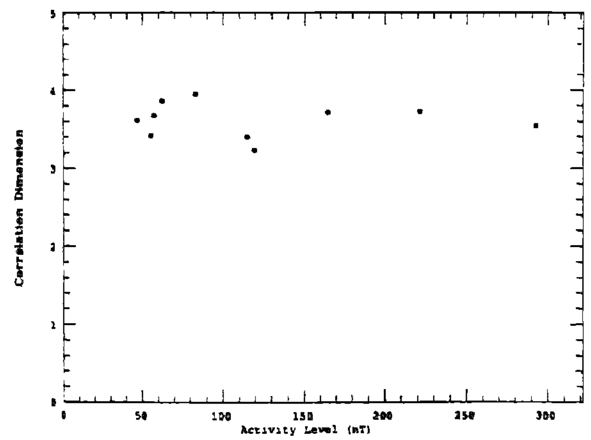

Fig. 3. The attractor dimension ν converges between 3 and 4 for all data segments and is plotted here versus an estimate of their activity level.

Another way to correct this is to shift the time interval until the activity is again sufficiently uniform.

The sensitivity of the correlation-dimension method to external noise was also tested. Such noise could be the output of a random process or poor measurement conditions. White noise was added to the original measurements and the dependence of the dimension on its level was measured. This is shown for two different sets of data (0–5 kmin and 21–27 kmin) in Figure 4; here the noise level has been normalized to the activity of each sample (83 nT and 292 nT, respectively). Initially until 3–4% of the activity level the dimension estimate stays almost constant, but then it increases at least linearly with noise. For a finite data set the rate of increase also depends on its length. The invariance of the dimension to small levels of noise is indicative of the accuracy of the method; similarly the sensitivity of the result to higher noise levels proves the quality of (low-noise) data.

The entropy K_2 as defined above was also computed for the segment 0–10 kmin (Figure 5). The lines are parametrized

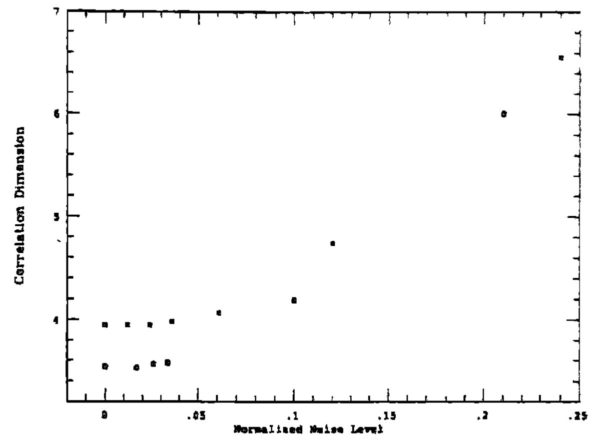

Fig. 4. Effect of noise of the dimension estimates for two segments with quite different activity levels (squares: 0–5kmin, circles: 21–27kmin). For each case the noise level is normalized to the activity level of the segment, 83 nT and 292 nT.

by r-values, which grow from top to bottom. For too low r's the statistics do not allow for good convergence; therefore only r-values above a chosen threshold are shown. The value found for the entropy, 0.2/min, has two important implications. A finite entropy suggests that the time series data does not represent a random system, but rather a chaotic one. Secondly the time scale characteristic of the system is ~5 mins.

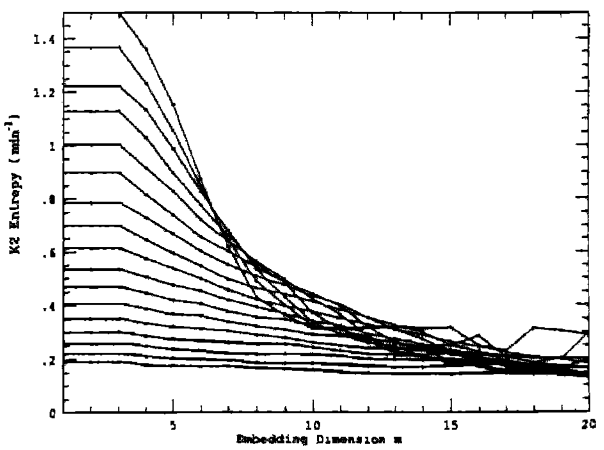

Fig. 5. Convergence of the entropy K_2 with the embedding dimension m from the first 10 kmin of the time series. The convergence value, 0.2 min^{-1}, is an estimate of the Kolmogorov entropy of the signal.

Summary and Discussion

Using the nonlinear methods outlined above, the state space of the magnetospheric response as described by the AE index was reconstructed from the time series data. In the reconstructed state space, the system is seen to evolve on a fractal set with dimension 3.6. This result suggests two things: first, the dynamics of the system is neither quasiperiodic nor random, but chaotic. The absence of quasiperiodic or strong random components is verified by tests using this method as well as by other diagnostics (e.g., the continuum nature of the power spectrum; the finite value, 0.2/min, of the K_2 entropy). Secondly, the low dimension indicates that the system can be described by four independent variables. By "system" we do not mean the global magnetosphere but only its response as represented by the AE index. Both findings encourage recent attempts [Baker et al., 1990] at constructing simple, deterministic models of the loading-unloading process in the magnetosphere that yield chaotic behavior for a broad range of parameters.

It is important, before closing, to address the relevance question. Are analyses such as the one presented here simple intellectual curiosities or can they lead to advances in understanding and modeling of the magnetosphere? At present the only honest answer is "we do not know but it is worth exploring". The fact that a low-dimensional system was identified in the study encourages us to proceed further. If a higher dimension had been found (say, $\nu>10$) the system would be best addressed by conventional statistical techniques. The next step in the analysis is to identify the appropriate variables and to infer possible forms for the system's evolution equation. This requires a combination of physical understanding along with nonlinear analysis. For the case considered here, we note that the AE index is a measure of overall geomagnetic activity in the auroral region and is derived from measurements of the north-south components of magnetic field fluctuations at a number of ground stations. These fluctuations are due to the electrojet current whose strength is proportional to the precipitation process and the strength of the parallel electric field. Further these currents lead to heating of the ionospheric plasma, thus affecting the magnetic and electric fields. From these considerations, a choice of the physical variables could be the north-south and vertical magnetic fields, the parallel electric field and the plasma temperature. We do not expect the solar wind to belong to this set of variables because it possesses features of a random process associated with a large number of degrees of freedom (indeed the corresponding dimension ν does not seem to converge to a small value). With this or another choice of variables, the techniques for the construction of equations of motion [Crutchfield and McNamara, 1987] may be used to construct a model whose phase space can then be compared to the one reconstructed from the AE data. This work is in progress and results from such an attempt will be presented in a future paper.

Acknowledgements. We acknowledge fruitful discussions with W. Marable, D. Baker, A. Klimas and A. Roberts. This work is supported by the NASA/ISTP grant NAG 5-1101.

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(Received June 25, 1990;
revised August 14, 1990;
accepted August 20, 1990.)